# Computer Vision 

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Test conditions: All documents and reference materials are authorized. You may NOT communicate with anyone other than the exam Proctor or the course professor. You must answer all questions in INK on the official exam paper. You may use scratch paper to prepare your answer, but your scratch paper will not be graded. You may respond in English or French (or both), but you MUST write legibly. Illegible text will not be graded. Use mathematics as well as English and/or French to communicate.

1) (4 points) Assume a camera at position $(0,0,2)$ and orientation $(-\pi / 2,0,0)$ with focal length $F$, equipped with a $512 \times 512$ pixel retina in which pixels are size $0.02(\mathrm{~mm} / \mathrm{col})$ et $0.01(\mathrm{~mm} / \mathrm{row})$ and an optical axis that intersects the retina at pixel $(256,256)$. This is described by a projective transformation $P^{i}=\mathbf{M}_{s}{ }^{i} P^{s}=\mathbf{C}_{r}^{i} \mathbf{P}_{\mathrm{c}}^{\mathrm{r}} \mathbf{T}_{\mathrm{s}}^{\mathrm{c}} \mathrm{P}^{\mathrm{s}}$. Write the matrices for the transformations $\mathbf{C}_{r}^{i}, \mathbf{P}_{\mathrm{c}}{ }^{r}$, and $\mathbf{T}_{\mathrm{S}}{ }^{\mathrm{c}}$ as well as for the camera projective matrix $\mathbf{M}_{\mathrm{S}}{ }^{i}$.
2) (12 points) We can use color statistics to build a simple detector for skin color regions
a) Explain how to transform RGB pixels into a two-component chrominance vector ( $\mathrm{c}_{1}, \mathrm{c}_{2}$ ) that is normalized to eliminate illumination intensity.
b) Explain how to use histograms of chrominance vectors to determine the probability that a pixel contains an image of skin.
c) Explain how to compute the probability that an image window of size IxJ contains a region of skin of size WxH based on skin-color probabilities (assume $\mathrm{W}<\mathrm{I}$ and $\mathrm{J}<\mathrm{H}$ ).
d) Explain how to compute the center of gravity of the skin color region from the probability of skin in the image window of size IxJ.
e) Explain how to compute the second moment of the skin color region from the probability of skin in the image window of size IxJ.
f) Explain how to determine the width, length and orientation of the skin color region from the second moment.
3) (4 points) You are asked to compute the gradient $\vec{\nabla} P(x, y)$ using convolution with sampled Gaussian derivatives, $\vec{\nabla} G(x, y)$ with $\sigma=2$.
a) Give the formulae for the sampled Gaussian derivatives as well as the formulae for the 2-D convolution with an image.
b) Show that the gradient $\vec{\nabla} P(x, y)$ can be computed as a sequence of convolutions with 1-D Gaussian derivatives in the row and column directions.
c) What is the computational cost in terms of additions and multiplications to compute the gradient $\vec{\nabla} P(x, y)$ using convolution with sampled Gaussian derivatives, $\vec{\nabla} G(x, y)$ with $\sigma=2$ when implemented as a 2D convolution?
d) What is the computational cost in terms of additions and multiplications to compute the gradient $\vec{\nabla} P(x, y)$ using convolution with sampled Gaussian derivatives, $\vec{\nabla} G(x, y)$ with $\sigma=2$ when implemented as convolution with separable 1-D components?
