

# Computer Vision

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Lesson 6

Exercises

For an image  $P(i,j)$ , the gradient  $\vec{\nabla}P(i,j)$  is a vector composed of the first derivatives in the row and column directions. In the following questions, assume that  $P(i,j)$  is a luminance (black and white) image of size 1024 x 1024 pixels with 8 bits per pixel.

- 1) You are asked to compute the gradient  $\vec{\nabla}P(i,j)$  using convolution with sampled Gaussian derivatives,  $\vec{\nabla}G(i,j)$ . Give the formulae for the sampled Gaussian derivatives as well as the formulae for the 2D convolution.
- 2) What is the minimum size support window that can be used for a Gaussian derivative with  $\sigma=2$ ? What is the computational cost for such a convolution in terms of additions and multiplications when implemented as a 2-D convolution?
- 3) Show that the sampled Gaussian derivatives are separable. That is, that convolution with the sampled Gaussian derivative can be implemented as a sequence of convolutions with 1-D filters in the row and column directions. What is the computational cost in terms of additions and multiplications for the convolution of the image with sampled Gaussian derivatives at  $\sigma=2$  when implemented as convolution with separable 1D components?
- 4) Show that a 1D Gaussian low pass filter with  $\sigma=2$  can be implemented as a series of convolutions with 1D Gaussian low pass filter with  $\sigma=1$ . How many convolutions are needed to compute a 1D Gaussian low pass filter with  $\sigma=2$  as a series of convolutions with a 1D Gaussian low pass filter with  $\sigma=1$ ? What is the computational cost of this series of convolutions?
- 5) Is it possible to implement the convolution with a 1-D Gaussian Derivative filter with  $\sigma=2$  as a series of convolutions with 1-D Gaussian filters? If yes, how? If no, why not?
- 6) Given the Gradient of the image  $\vec{\nabla}P(i,j)$  in the row and column directions, give a formulae to determine the gradient at pixel  $(i,j)$  in an arbitrary direction  $\theta$ .
- 7) Given the Gradient of the image  $\vec{\nabla}P(i,j)$ , give the formulae to determine the direction of maximum gradient,  $\theta_{\max}(i,j)$  at each pixel  $i,j$ .
- 8) Give a formula for the coefficients for a binomial low pass filter at  $\sigma=2$ .
- 9) Show that a binomial low pass filter at  $\sigma=2$  can be implemented as a series of convolutions with the binomial filter  $[1 \ 2 \ 1]$ . How many such convolutions are required?
- 10) Assume that you use  $[1 \ 0 \ -1]$  as a first derivative operator. What is the computational cost (additions and multiplications) for computing the Gradient of an image at  $\sigma=2$  using a series of convolutions with 1-D binomial filters  $[1 \ 2 \ 1]$  in the row and column directions?